Eighth Grade: Mathematics
Unit #1: Exploring Linear and Inverse Relationships

Overarching Question:
What does it mean when we see constant and predictable changes in a table of data or a graph?

Questions to Focus Assessment and Instruction:
1. What do the slope and y-intercept of a line represent in a real-world situation?
2. How can data in a table or scatterplot be used to predict a future outcome?
3. How can you recognize linear and non-linear relationships between two variables in a table? in a graph? in an equation?
4. How can you solve an equation using a table? using a graph? using symbolic manipulation?
5. How does change in one variable affect change in the other variable in a linear relationship?

Intellectual Processes (Standards for Mathematical Practice)

Use appropriate tools strategically:
Use graphing tools to model linear change in tables, graphs, and equations.

Make sense of problems and persevere in solving them:
Strategically choose and efficiently implement procedures to solve linear equations.

Model with mathematics:
Identify proportional relationships and make connections to linear functions.

Key Concepts:
- rate of change
- initial value
- slope
- direct variation
- constant of proportionality
- proportional relationship
- unit rate
- x- and y-intercept
- scatterplot
- line of best fit
- solution
- inverse variation
- function
- scatterplot
- line of best fit
- solution
- inverse variation
- function
Mathematics
Exploring Linear and Inverse Relationships

Unit Abstract
Students generalize their knowledge of proportional situations to linear relationships and connect the constant of proportionality seen in tables and graphs to the rate of change (slope). They also represent linear relationships using tables, equations, and graphs. They write equations in \( y = mx + b \) form, and learn how the slope \( (m) \) portrays the relationship between variables. Scatterplots of collected data are used determine the relationship between the variables. In the case of a linear-like relationship, a line of best fit is applied. Students will represent linear functions by using spreadsheets and using graphing calculators or computer software. Questions about linear relationships in real-world and mathematical contexts are solved using a variety of methods including tables, graphs, and equations. Linear functions are contrasted with directly proportional patterns of change and inverse relations.

Common Core State Standards

Expressions and Equations (8.EE)
Understand the connections between proportional relationships, lines, and linear equations.

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
6. Use similar triangles to explain why the slope \( m \) is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation \( y = mx \) for a line through the origin and the equation \( y = mx + b \) for a line intercepting the vertical axis at \( b \).

Analyze and solve linear equations and pairs of simultaneous linear equations.

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form \( x = a \), \( a = a \), or \( a = b \) results (where \( a \) and \( b \) are different numbers).
   b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

Functions (8.F)
Define, evaluate, and compare functions.

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.
3. Interpret the equation \( y = mx + b \) as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function \( A = s^2 \) giving the area of a square as a function of its side length is not linear because its graph contains the points \((1, 1), (2, 4)\) and \((3, 9)\), which are not on a straight line.*

*a. Include inverse variation relationships as examples of functions that are not linear. Write equations of inverse relationships and use them along with graphs, tables, and verbal descriptions to solve problems, make predictions and decisions.

Use functions to model relationships between quantities.

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two \((x, y)\) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**Statistics and Probability (8.SP)**

Investigate patterns of association in bivariate data.

2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. *For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.*

* Indicates items added to CCSS

**Instructional Resources**

NCTM Illuminations ([http://illuminations.nctm.org](http://illuminations.nctm.org))

**Equations of Attack:** When one end of a wooden board is placed on a bathroom scale and the other end is suspended on a textbook, students can "walk the plank" and record the weight measurement as their distance from the scale changes. The results are unexpected— the relationship between the weight and distance is linear, and all lines have the same x-intercept. This investigation leads to a real world occurrence of negative slope, examples of which are often hard to find.


**Amazing Profit:** Students use equations to determine eBay profit on new technology. EBay is an online auction agency. For a limited time after a “new” product’s street release date, it is possible to track the profit that sellers make for auctioning them on eBay. Students use previous data of selling prices to derive a linear equation for the “closing bid price” on a product.
http://illuminations.nctm.org/LessonDetail.aspx?id=L799

**Finding Our Top Speed:** This lesson sets the stage for a discussion of travel in the solar system. By considering a real-world, hands-on activity, students develop their understanding of time and distance. Finally, students plot the data they have collected.

http://illuminations.nctm.org/LessonDetail.aspx?id=L254

**Bouncing Ball:** Students develop their skills in collecting and recording data using the real-world situation of a bouncing tennis ball. They use the data collected to formulate the relationship between the dependent and independent variable in their experiment.

http://illuminations.nctm.org/LessonDetail.aspx?id=L246

**Exploring Linear Data:** Students model linear data in a variety of settings that range from car repair costs to sports to medicine. Students work to construct scatterplots, interpret data points and trends, and investigate the notion of line of best fit.

http://illuminations.nctm.org/LessonDetail.aspx?id=L298

**Texas Instruments**

**Discount Savings (TI-73):** In this activity, students will find discounts and sale prices for items selling at 20% off and 40% off by using tables. They will then find the general rules, using a variable, for finding those discounts and prices.


**Growing the Green (TI-73):** Students will explore percentage increase over time and consider the power of compounding using percents.

http://education.ti.com/calculators/downloads/US/Activities/Detail?id=8455

**Geogebra**

Proportions in Similar Triangles

**Explorelearning, Gizmos! (http://www.explorelearning.com)**

**Points, Lines, and Equations:** Compare the graph of a linear function to its rule and to a table of its values. Change the function by dragging two points on the line. Examine how the rule and table change.


**Direct Variation:** Adjust the constant of variation and explore how the graph of the direct variation function changes in response


**Slope:** Compare the slope-intercept form of a linear equation to its graph. Find the slope of the line using a right triangle on the graph. Vary the coefficients and explore how the graph changes in response.


**TEXTEAMS Rethinking Middle School Mathematics: Proportionality**
Here is an example of one of the activities available:

**Jet Ski Rental:** Distinguish between a proportional and non-proportional situation using the characteristics of a proportional relationship.

**Short Stack:** Investigate proportional and non-proportional relationships by measuring the heights of stacked objects.

**Thinking Rationally about Fractions, Decimals, and Percent: Instructional Activities for Grades 4 through 8:** This resource, developed by the Virginia Department of Education, has a large number of activities and lessons to provide instruction to middle grades students in fractions, decimals and percent.


**Other Resources**

**Calculation Nation:** The games of Calculation Nation® are organized around content from the upper elementary and middle grades math curriculum. By becoming a citizen of Calculation Nation®, your child or student will play online math strategy games that allow them to learn about fractions, factors, multiples, symmetry and more, as well as practice important skills like basic multiplication and calculating area — all while having fun.

http://calculationnation.nctm.org/

**Figure This!** Math Challenges for Families has a variety of problems for students to solve. There is a Math Index, which sorts the problems by math content.

http://www.figurethis.org/index.html

**Weekly Problems for Middle School Students: Past Problems and Solutions**

http://www.nctm.org/resources/archive.aspx?id=3604&journalid=3

**Proportional Reasoning Math Skills- Scale City:** This site is for grades 6-8 and provides a variety of fun activities using proportional reasoning. Each activity provides an entertaining video of real-life uses for proportions related to the name of the activity. There is also an assessment covering all the ideas in the site. Activities include: Dinosaur World (one-dimensional scaling), World of Mural Painting (two dimensional scaling), Miniature Land (three dimensional scaling), World Chicken Festival (scaling recipes), Louisville Slugger Museum (Similar Triangles), Sky-Vue Drive-In (Introducing Inverse Proportions), Belle of Louisville (proportions and Music), Kentucky Horse park (Speed, Distance, and Time)

http://www.ket.org/scalecity/index.html

This applet allows students to explore how change the values of slope and y-intercept change the graph of a linear equation.

http://www.shodor.org/interactivate/activities/SlopeSlider/

An online lesson introducing the lines and slope.

http://mathforum.org/cgraph/cslope/
Mathematics
Exploring Linear and Inverse Relationships

Students pass a "hand squeeze" around a circle and measure the amount of time that it takes for the hand squeeze to pass around the circle.
http://math.rice.edu/~lanius/Algebra/hndsq.html

Assessment
Interactive Algebra Quiz
http://www.shodor.org/interactivate/activities/AlgebraQuiz/

Professional Resources

NCTM (www.nctm.org)
Yearbook: NCTM's 2002 Yearbook emphasizes that although fractions, ratios, and proportions are pivotal concepts in the middle school, their development and understandings begin in the elementary school. The companion booklet presents activities that illustrate some of the ideas in the yearbook and that go beyond the content of the yearbook itself. Teachers' notes and handouts are designed to bring the yearbook to life in the classroom.

Essential Understanding Series: Developing Essential Understanding of Ratios, Proportions, and Proportional Reasoning for Teaching Mathematics: Grades 6-8. This book goes beyond a simple introduction to ratios, proportions, and proportional reasoning. It will help broaden and deepen your mathematical understanding of one of the most challenging topics for students.

Navigating through Algebra in Grades 6–8: This book shows how middle school students can use mathematical models and represent and analyze mathematical situations and structures to explore the concept of function. The activities and problems require students to use representations related to work with functions, and they highlight some of the interactions that may occur among these representations.

Articles from National Council of Teachers of Mathematics (www.nctm.org)
Articles are available as free downloads to NCTM members, or for a fee to non-members.


http://www.nctm.org/eresources/article_summary.asp?from=B&uri=MT2000-12-762a